Vibration Analysis of Multi Degree of Freedom Self-excited Systems

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Abstract— It is known that nonlinearities in the friction-velocity curves of dry friction may lead to instability of the steady frictional sliding, namely, friction-induced vibration. Therefore, stick-slip vibration is an important issue in mechanical engineering, and understanding the dynamics of stick-slip and its possible elimination also becomes important, especially for applications requiring high-precision motion. In this paper, multi-degree of freedom self-excited vibrating systems which are excited by the friction force imposed from a moving surface will be considered. The vibration response and velocity of system’s components under various initial conditions will be calculated and reported. The effect of initial conditions, velocity of moving surface and its coefficient of friction will be studied and discussed. The result of this study provides an understandable structure for vibro-acoustical analysis of self-excited vibrating systems.

Keywords—Friction Contact, Friction-Induced Vibrations, Self-Excited systems, Brake Noise, Non-linear Time-Series Analysis, Chaotic Dynamics.

I. INTRODUCTION

The main cause of self-excited vibration phenomena is actually the excitation imposed by friction between the mechanical parts of the system. Leonardo da Vinci was the first person who formulated the rules of friction. He hypothesized that friction might be independent of the area of contact and there is a direct connection between the applied normal load and the vibration frictional force. Also, Coulomb, in 1781, realized a new concept of a limiting static friction. He believed that this friction would not prevent static bodies from movements unless the advancing force exceeds this friction. Specifically, he stated that this amount is greater than the coefficient of kinetic friction. These ideas and the concept that the frictional force is almost independent of the sliding speed are considered as Coulomb’s friction law. Because of several objectives the Coulomb’s friction law reminisced as a so competent pattern for friction.[1]

Recently, the ability of prediction and reduction of structural vibrations has received more attention. The origins of vibrations are either from external sources, e.g. wind, or internal ones like friction between mechanical parts of system. The main example for self-excited vibration is actually break squeal in automobiles. It can change the braking ability of car which is actually undesirable. In fact, the reliability of braking system in cars plays an important role in the safety of passengers. Therefore, researchers put a lot of efforts on the study of self-excited vibration phenomena and its applications in the real life. Self-excited vibrations become evident because of a mechanism of a system which will vibrate at its own natural or critical frequency spontaneously, the amplitude increasing until some nonlinear effect limits any further increase and their oscillation has been known as one of the cases of vibrations that may happen either with or without external or internal periodic forcing. Their occurrence is fundamentally depends on the special internal features of a system. From a mathematical aspect, the motion can characterize with the unstable homogeneous solution to the homogeneous equations of motion. Furthermore, in the forced or resonant vibrations, the fluctuation frequency is dependent on the frequency of a forcing function which is act as an external exciter to the system. In nonhomogeneous equations of motion the particular solution is forced vibration.

Self-excited vibrations have pervaded in different parts of physical systems that motion or time-variant factors are included. Instance of these parts are: aerothermodynamics, aerodynamics, aeromechanical systems, feedback networks, and mechanical systems [2]. Vibrations of a vehicle wheel, unwanted vibrations during machining processes, vibration of airplane wings are some classical examples of self-excited systems. The happening of self-excited vibration in a physical system is connected with the
stability of equilibrium positions of the system. If system's equilibrium disturbed, some forces emerge that lead the system to move toward or away from its equilibrium. In forced vibration system the equilibrium position is unstable therefore it may either monotonically recede from the equilibrium position until nonlinear or limiting restraints appear or oscillate with increasing amplitude. In both of the cases if the disturbed system approaches the equilibrium position either, the equilibrium is been considered as stable.

When system is disturbed and moved away from its equilibrium depending upon the displacement of the velocity the appearance of the forces change. If displacement-dependent forces become manifest and cause the system to move away from the equilibrium position, the system is said to be statically unstable. Moreover, velocity-dependent forces that lead system to withdraw from a statically stable equilibrium position results in a dynamic instability.

It is seems that the instability of the steady frictional sliding, known as friction-induced vibration, is initiated from the nonlinearities in the friction-velocity curves of dry friction. Famous examples of such friction-induced vibrations are the motion of a violin and Froude pendulum. However, the affliction inclination of the sliding components to stick and slip causes to some problems in industrial usages. For instance, in the operations such as robot joints, electric motor drives, disk brake systems, bearings, machine tools and work piece systems, wheel and rail mass transit systems, etc. it may have some negative effects. Hence, comprehension of stick-slip vibration with the hope of its elimination in industrial applications, especially in high precision motions, has recently caught more attention [3].

In the next chapter, a lecture survey will be done to understand the background of self-excited vibration analysis in systems.

In 1938, Jarvis and Mills [4] showed the relationship between the relative velocity of brake disk and wheel speed with respect to self-excited vibration aspects. It was indicated that self-excited vibration which is induced by the friction in the brake system of car, can reduce the braking instability of car. They indicated that the alteration of the friction coefficient by sliding speed was inadequate to lead to friction-induced vibrations thus the instability was depend to coupling although the friction coefficient was constant.

A comprehensive analysis power for identification of the influences of physical parameters on the stability of system is needed in order to solve friction-induced difficulties. As an illustration, the friction coefficient has some influences on the stability of static solution. These various mechanisms should be considered to gain knowledge an appropriate mechanism to clarify friction induced oscillation in systems. It was discovered that the self-excited vibration may happen while the friction coefficient stayed constant with speed. Spurr [5] presented the sprig-slip phenomenon that doesn't rely on a friction coefficient changing by the relative rotation speed of the brake disk. Later scholars [6-11] improved a more generalized theory explaining the mechanism as geometrically induced or kinematic constraint instability.

Although, there is a lack of uniform theory for modeling of the problem and that sprig–slip phenomenon, but there are several publications which address the mechanism of dynamic instability of brake systems by the stick-slip phenomena [12], geometric coupling of the structure involving sliding parts [4,5,7,9,13-15] and negative friction velocity slope [16,17].

The fluctuation of a system that consists of a mass and spring which sliding on a moving belt can simulate the active control system. Generally, two kinds of friction-induced vibrations are known: the first one exhibits a sinusoidal wave with its frequency near the natural frequency. It is produced by the negative of friction coefficient on relative velocity which acts as a negative damping [18]. So, in order to make the relationship positive, in the design stage of practical frictional surface, some materials and lubricants have been developed to make this inversion. The second kind of friction-induced vibration has a triangular wave or a saw tooth wave that its frequency is depend on the sliding velocity. Due to the fact that this vibration is produced by the repetition of “stick” and “slip” of mating surfaces it is known as stick-slip motion [19]. The difference among the static and kinetic friction coefficient lead to the Stick-slip motion that is inclined to emerge in higher normal load and lower sliding velocity. [20]

Self-excited vibration has a several differences with forced vibration: in self-excited vibration the alternating force that nourish the motion is produced by the motion itself and it disappears when the motion cease. However, in the forced vibration systems the nourishing alternating force is autonomous of the motion and survives even if the vibratory motion is disappeared. [21]

There are two primary types of self-excitation, i.e., hard self-excitation characterized using an unstable limit cycle, and soft self-excitation symbolized by stable limit cycle. The trajectory in the hard self-excitement, according to the initial situation inclines to infinity or an equilibrium point. [22]

Differential equations with time delay terms or nonlinear functions of state space coordinates could be applied to exhibit the mechanism of self-excitation. It should be noted that the mathematical models of the self-excited systems instead of containing direct time elements is controlled by independent differential equations which is
friction-induced fluctuations. Finally the origin and the role of phase shifts between oscillations normal and parallel to the contact surface is clarified with respect to the mode-coupling instability.

Hoffmann and Gaul [35] worked on the consequence of damping on destabilizing of mode-coupling in self-excited friction induced fluctuation. Furthermore they determine that the destabilization of friction-induced fluctuations may occur by increasing damping and also noticed that they can’t relinquish the side effect simply.

The “mass-on-moving-belt” model for presenting friction-induced vibrations was introduced by Thomson et al. [36]. It was described that the friction forces in two phases of first decreasing and then increasing smoothly through relative interface speed. Close analytical descriptions were shown for the conditions, amplitudes and the base frequencies of friction-induced stick-slip and pure-slip oscillations.

Chen et al. [37] introduced a self-excited system which depends on the time lag among the normal force oscillation and a friction oscillation that created from it. A number of simulations were performed by using the model. The consequence of his model indicates that instability vibration can be excited by the time delay.

In the field of controlling the friction that lead to self-excited vibration Chatterjee [38] presented a new method. The system model is shown by a single degree of freedom oscillator on a moving belt. The control action was set by adjusting the normal load at the frictional interface based on the state of the oscillatory system.

Ranjar et al. investigated the dynamical behaviours of various structures in their studies. He has worked in this field for more than ten years. A series of his publications can be found in ref. [39-59].

After performing a careful study on the history of vibration analysis of self-excited system, it has been followed that there are several open question still available in this field, e.g. behavior of system under self-excited condition and proposed noise from a self-excited vibratory system. Therefore in this paper, a comprehensive study will be performed to understand more about the behavior of multi-dimensional self-excited systems. Chapter three discusses about the research methodology that is used in this study and related data.

II. MECHANICAL MODEL

The modelling of mass-spring system with two-degree-of-freedom is presented in this section.

The equation of motion can be expressed as

$$M \ddot{X}(t) + C \dot{X}(t) + KX(t) = F(t)$$  (1)
The two-degree-of-freedom model illustrate in fig. 1, as it shown this system consist of two masses on a moving belt and three springs that two of them fix the masses to block.

![Diagram of a two-degree-of-freedom system with two masses and springs](image)

**Fig. 1** Self-excited (friction driven) system with two degree of freedom.

Typically, for solving the vibration problems we set the motion equation in the matrix form as:

\[
\begin{bmatrix}
m_1 & 0 \\
0 & m_2 \\
\end{bmatrix} \begin{bmatrix}
x_1(t) \\
x_2(t) \\
\end{bmatrix} + \begin{bmatrix}
k_1 + k_2 & -k_2 \\
-k_2 & k_1 + k_3 \\
\end{bmatrix} \begin{bmatrix}
x_1(t) \\
x_2(t) \\
\end{bmatrix} = \begin{bmatrix}
f_1 \\
f_2 \\
\end{bmatrix}
\]

(2)

while assumed initial conditions for the system are

\[
x(0) = \begin{bmatrix}
0.05 \\
0.06 \\
\end{bmatrix} (\text{m}) , \quad x(0) = \begin{bmatrix}
0.04 \\
0.03 \\
\end{bmatrix} \text{ (s)}
\]

(3)

The Coulomb friction model states that the frictional force is independent of the magnitude of the velocity of [1] and it can be written as:

\[
f_i = -\text{sign}(x_i(t) - v_{\text{surf}}) \mu_i mg , \quad i = 1, 2
\]

(4)

where \(x(t)\) is the velocity of the body, \(v_{\text{surf}}\) is the velocity of surface and \(f\) is the friction force which is acting to the body, \(m\) is the mass, \(\mu_i\) is the dry friction coefficient, \(k\) is the spring stiffness coefficient, and \(g\) is the gravity acceleration. Although, it is indicated that the friction is independent of the magnitude of the velocity, but experiments have shown that it is velocity dependent. Lindop and Jensen [60] demonstrated this argument computationally based on a qualitative understanding of surface interactions.

To simulate the response of the spring-mass system located on a friction belt driven at some nonzero velocity, the initial value problem should be written. At this regard, we reduce the order of system by the replacing the original variable \(x\) with \(y\) as:

\[
y_1(t) = x_1(t) , \quad y_2(t) = x_2(t)
\]

(5)

\[
y_2(t) = x_1(t) , \quad y_4(t) = x_2(t)
\]

Then, we reformulate the Eq. 1 to an initial value problem as:

\[
\begin{bmatrix}
m_1 & 0 \\
0 & m_2 \\
\end{bmatrix} \begin{bmatrix}
y_2 \\
y_4 \\
\end{bmatrix} + \begin{bmatrix}
k_1 + k_2 & -k_2 \\
-k_2 & k_1 + k_3 \\
\end{bmatrix} \begin{bmatrix}
y_1 \\
y_3 \\
\end{bmatrix} = \begin{bmatrix}
f_1 \\
f_2 \\
\end{bmatrix}
\]

(6)

\[
y_2 = -m_1 g \text{sign}(v_1 - v_{\text{surf}}) + k_1 x_1 - (k_1 + k_2) x_2
\]

(7)

\[
y_4 = -m_2 g \text{sign}(v_2 - v_{\text{surf}}) + k_2 x_1 - (k_2 + k_3) x_2
\]

consider the masses \(m_1\) and \(m_2\) as one kilogram, the dry friction constant as 0.05, all spring stiffness constants as equal as 10 N/m, the speed of belt as 0.01 m/s and the gravitational acceleration as 9.81 m/s². The system is assumed to be initially at rest. Then by substituting of these values in Eq. 4 and changing the matrix form to a system of algebraic equations, we will have four algebraic equations as:

\[
\begin{bmatrix}
y_1 \\
y_2 \\
y_3 \\
y_4 \\
\end{bmatrix} = \begin{bmatrix}
y_2 \\
f_1 + y_3 + y_4 - 2 \times (y_1 + y_2) \\
fy_3 \\
f_2 + y_1 + y_2 - 2 \times (y_2 + y_3) \\
\end{bmatrix}
\]

(8)

with the initial conditions of

\[
\begin{bmatrix}
y_1(0) \\
y_2(0) \\
y_3(0) \\
y_4(0) \\
\end{bmatrix} = \begin{bmatrix}
0.05 \\
0.04 \\
0.06 \\
0.03 \\
\end{bmatrix}
\]

(9)

We can see the advertised self-excited vibrations when the system starts by carrying along the masses on the supporting surface with a specific pulling period. Then the oscillations start. Also, we may observe a bit of noise in the
initial stage when the pull on the surface is not get sufficiently strong, and the mass should be carried along at a constant speed. By solving of initial value problem indicated in the equations 6 and 7, we can calculate the vibration responses of both masses and their oscillating velocities.

A mathematical model is prepared to calculate the vibration responses of masses and their oscillating velocities. These results will help the designer and researchers to obtain perception to the interaction of system dynamics and friction and recognize the main parameters that influence on the system response. The next chapter reports the results of these models.

### III. RESULTS

In figures 2 and 3, the vibration response and oscillating velocity of masses for a system with two-degree-of-freedom are shown.

Figure 2.a shows the behaviour of mass \( m_1 \) when no surface friction is considered. It is clear that in this case, the system oscillate freely with a constant amplitude and period. Furthermore, figure 3.b presents the response of system when a coefficient of moving surface friction of 0.05 is considered for mass \( m_1 \).

It can be seen that due to effect of friction, the vibration of system is damped in 1.35 seconds and it will remain in its static position.

The amplitude of vibration for mass \( m_1 \) in figure 2.a is 0.06 m. Also, the period of vibration in this case is 2 seconds. Furthermore, in figure 2.b, it is shown that the mass \( m_1 \) will stick to the supporting surface and remain there after some initial movement due to initial moving condition.

In figure 3, Vibration response and velocity of mass \( m_2 \) with given initial conditions and static supporting surface are shown. Although the behaviour of mass \( m_2 \) in both cases of (a) and (b) are very similar with mass \( m_1 \) but obviously the amplitude of its free vibration case is smaller, i.e., 0.16 meters. Also, its vibration under the effect of friction force is stopped after 0.85 seconds (figure 3.b).

In the stick period there is a static force which controls the motion and in the slip period the velocity dependent on the dynamic friction. In the stick-slip phenomenon, stick occurs because the surfaces static friction is great and the slip happens caused by the low dynamic friction within sliding. By increasing the belt velocity at a specific velocity the stick-slip phenomenon ceases to exist. Meanwhile, decreasing of surface velocity without considering an appropriate amount for coefficient of friction can cause the masses to stick onto surface of moving belt.
IV. CONCLUSION AND FUTURE WORK

The friction coefficient has a significant effect in the stability of a system. Thus in this paper, the self-excited vibration of a system with two degree of freedom and a system with three degree of freedom was represented. There are factors that changing the values of them, affect the behavior of a system and also stick-slip phenomenon. Hence these results should be informed to cause a reduction or generation in a brake noise.

It is necessary to perform a stability analysis for the system to understand in which values of system parameter, it is oscillating or sticking. Increasing the friction coefficient reduce the number of oscillations and decreasing it lead a decrease in stick-slip oscillation (noise) but the problem is it lessen the brake efficiency by the way. It was shown that by decreasing the damping coefficient the stick-slip oscillation decrease and the system tend to damped. We also demonstrated that increasing the mass value makes the sticking period longer and the system tends to quasi-harmonic oscillations while increasing the spring stiffness cause to decrease the amplitude and stick-slip oscillation. Last but not least, the role of boundary condition is significant and we can’t disregard the effect of it on producing the stick-slip phenomenon.

As the future work, other suggested model for the approximation of friction force should be investigated. Also approximation of friction force for complex system, where there is no analytical formulation available for the models of friction force, should be studied. At this regard, such models like friction of two discs i.e. squeal phenomenon and other shapes can be considered.

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